

Analysis of Heat Transfer in the Entrance Region with
Fully Developed Turbulent Flow between Parallel Plates
—— The Case of Uniform Wall Heat Flux ——

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The equation describing the turbulent Graetz problem (heat transfer to a fluid between parallel plates) was solved numerically for the first ten eigenvalues and constants in the range of both $0.01 \leq Pr \leq 10$ and $10^4 \leq Re \leq 5 \times 10^5$.

The latest correlations of turbulent fluid velocity, eddy diffusivity and turbulent Prandtl number were used in the calculations. The numerical solution was obtained in much the same way as was previously done for the case of uniform wall temperature. The effect of the uniform wall heat flux boundary condition on heat transfer rate is elaborated.

INTRODUCTION

A number of papers have appeared on turbulent heat transfer in the thermal entrance region between parallel plates in connection with advances of nuclear reactors and gas turbine regenerators. However, the fluid velocity, eddy diffusivity and turbulent Prandtl number distributions used by them have not necessarily agreed with the available experimental data. In particular, most of these papers lack in a suitable discussion of the turbulent Prandtl number. Such discussion is barely covered by Hatton et al.¹⁾ and Larson et al.²⁾ for the case of uniform wall heat flux.

For example, Hatton et al. solved the eigenvalue heat transfer problem for uniform heat flux on one wall and the other wall insulated, but the eigenvalues and constants presented by them were

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only the first four terms. In uniform wall mass flux, Larson et al. showed that the experimental results agreed well with a finite difference solution to the turbulent diffusion equation, however, they represented only the numerical example for $Sc=0.62$.

The purpose of present study is to obtain a more accurate solution to the eigenvalue heat transfer problem between parallel plates in the case of uniform wall heat flux. For the case of uniform wall temperature, the solutions analyzed by Shibani et al.³⁾ and Sakakibara et al.⁴⁾ are comprehensive. In the calculations, we review previous studies on the fluid velocity, eddy diffusivity and turbulent Prandtl number distributions, and use suitable correlations for them. The equation describing the turbulent Graetz problem is solved numerically for the first ten eigenvalues and constants in the range of $Pr=10, 0.7, 0.1, 0.01$ and $Re=10000, 50000, 100000, 500000$. The boundary conditions are uniform heat flux on one wall, the other wall insulated (asymmetry) and uniform heat fluxes on each wall (symmetry). Furthermore, this numerical study is compared with the experimental data to evaluate the reasonableness of the results.

ANALYSIS

Basic Equation

Consider heat transfer to an incompressible fluid flowing in steady, fully developed, turbulent flow between parallel plates with their walls kept at uniform heat flux on one wall and the other wall insulated. The fluid enters the channel at a uniform and constant temperature t_e . If the energy dissipation is small and if the axial conduction can be neglected, then the energy equation is

$$u \frac{\partial t}{\partial x} = \frac{\partial}{\partial y} [(\alpha + \epsilon_h) \frac{\partial t}{\partial y}] \quad (1)$$

with the boundary conditions

$$\begin{aligned} x \leq 0 : \quad t &= t_e \\ x > 0 : \quad y &= 0, \quad \partial t / \partial y = -q/k \\ y &= 2y_0, \quad \partial t / \partial y = 0 \end{aligned} \quad (2)$$

The dimensionless variables are introduced as

$$y^* = \frac{y}{y_0} = \frac{y^+}{y_0^+}, \quad u^* = \frac{u}{V} = \frac{u^+}{V^+}, \quad x^* = \frac{x/D_e}{RePr}, \quad \theta = \frac{t - t_e}{qD_e/k}$$

where $D_e = 4y_0$

The solution is obtained in two parts

$$\theta = \theta_1 + \theta_2 \quad (4)$$

where θ_1 is the fully developed temperature profile and θ_2 is the entrance region temperature profile. At a large distance downstream of the thermal entrance, θ_2 must approach zero.

The solution θ_1 from a simple heat balance is given by

$$\theta_1 = 2x^* + H(y^*) \quad (5)$$

The differential equation for $H(y^*)$ is found to be

$$\frac{d}{dy^*} \left[\gamma \frac{dH}{dy^*} \right] = \frac{u^*}{8} \quad (6)$$

where $\gamma = 1 + \text{Pr} \left(\frac{\epsilon h}{\nu} \right)$

subject to the boundary conditions

$$\begin{aligned} y^* = 0, \quad dH/dy^* &= -1/4 \\ y^* = 2, \quad dH/dy^* &= 0 \end{aligned} \quad (7)$$

The solution for the entrance region distribution θ_2 is obtained by the method of separation of variables.

$$\theta_2 = \sum_{m=0}^{\infty} C_m Y_m(y^*) \exp(-\lambda_m^2 x^*) \quad (8)$$

where λ_m , Y_m are m th eigenvalue and eigenfunction of Sturm-Liouville problem, respectively. The eigenfunction is given by

$$\frac{d}{dy^*} \left[\gamma \frac{dY_m}{dy^*} \right] + \frac{\lambda_m^2 u^* Y_m}{16} = 0 \quad (9)$$

with the boundary conditions

$$\begin{aligned} y^* = 0, \quad dY_m/dy^* &= 0 \\ y^* = 2, \quad dY_m/dy^* &= 0 \end{aligned} \quad (10)$$

The coefficient C_m is given by

$$C_m = \frac{\int_0^2 (-H) u^* Y_m dy^*}{\int_0^2 u^* Y_m^2 dy^*} \quad (11)$$

The complete temperature distribution and the local Nusselt number are given by Eqs. (12) and (13), respectively.

$$\theta = 2x^* + H(y^*) + \sum_{m=0}^{\infty} C_m Y_m(y^*) \exp(-\lambda_m^2 x^*) \quad (12)$$

$$\text{Nu} = \frac{1}{H_1 \left[1 - \sum_{m=0}^{\infty} C_m \exp(-\lambda_m^2 x^*) \right]} \quad (\text{asymmetry}) \quad (13)$$

The local Nusselt number for uniform heat fluxes on each wall can be derived from superposition of temperature profile due to

a unit heat input.

$$\text{Nu} = \frac{1}{H_i \left[1 - \sum_{m=0}^{\infty} C_m \exp(-\lambda_m^2 x^*) \right] + \left[H_j - H_i \sum_{m=0}^{\infty} C_m (-1)^{m+1} \exp(-\lambda_m^2 x^*) \right]} \quad (14)$$

(symmetry)

Fluid Velocity and Eddy Diffusivity Distributions

Figs. 1 and 2 show typical expressions and experimental values for the fluid velocity and eddy diffusivity distributions between parallel plates. In Figs. 1 and 2, the experimental values of the fluid velocity distribution are larger than the values of Deissler's expression modified by Hatton et al.⁵⁾, while those of the eddy diffusivity distribution tend to be smaller. From the available experimental works illustrated in Figs. 1 and 2, the fluid velocity and eddy diffusivity distributions due to Mizushina et al.⁶⁾ [see Appendix I] show the best agreement over a wide range of Reynolds number. Therefore, their expressions were used to solve the energy equation in this work.

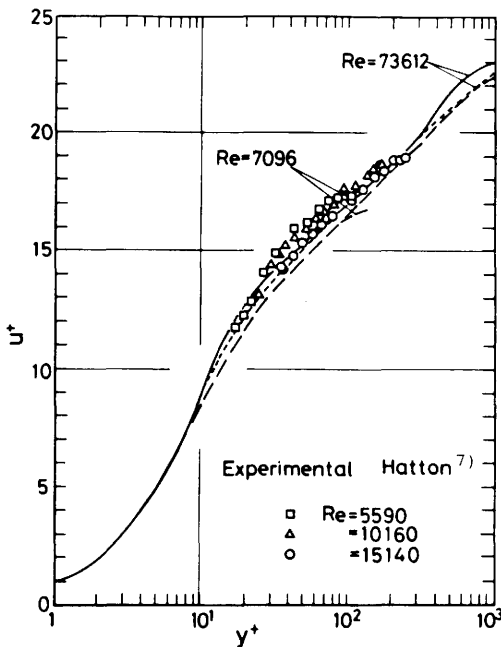


Fig. 1 Universal velocity profiles [——— Mizushina et al.⁶⁾; - - - Spalding⁸⁾; - - - Hatton et al.⁵⁾]

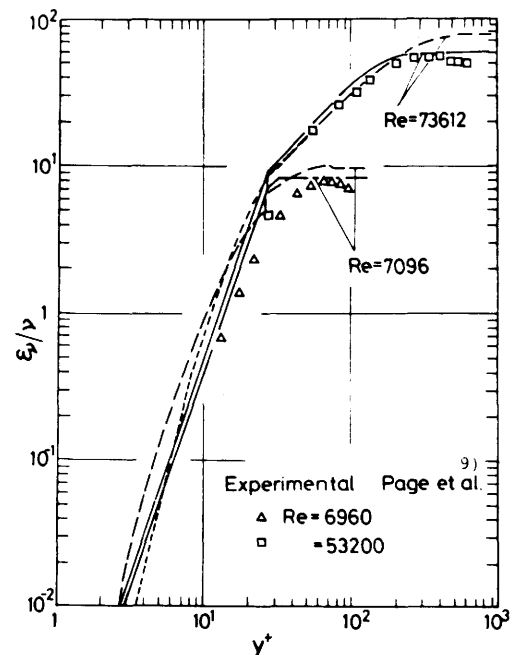


Fig. 2 Distributions of eddy diffusivities [——— Mizushina et al.⁶⁾; - - - Spalding⁸⁾; - · - · - Hatton et al.⁵⁾]

Turbulent Prandtl Number

The turbulent Prandtl number, $Pr_t = \epsilon_v / \epsilon_h$, is an important parameter for turbulent flow transport, yet there has been no rigorous theory offered on this subject [see Quarmby et al.¹⁰⁾]. We have made evaluations based on the available experimental data and have used Eqs. (15)¹¹⁾, (16)²⁾ and (17)¹²⁾ which are thought to be the most reasonable as far as the Prandtl number is concerned.

$$Pr = 10$$

$$Pr_t = \frac{1}{0.1265y^* + 1.064} \quad (15)$$

$$Pr = 0.7$$

$$Pr_t = 0.86 \quad (16)$$

$$Pr = 0.1 \text{ and } 0.01$$

$$Pr_t = \frac{[1 + 90Pr^{3/2}(\epsilon_v/\nu)^{1/4}][35 + (\epsilon_v/\nu)]}{[0.025Pr(\epsilon_v/\nu) + 90Pr^{3/2}(\epsilon_v/\nu)^{1/4}][45 + (\epsilon_v/\nu)]} \quad (17)$$

where $0 \leq y^* \leq 1$

RESULTS AND DISCUSSION

Analytical Results

For the calculation in Eqs. (6) and (9), it is necessary to add the over-all heat balance equation in Eq. (6) and the condition $Y = -H_i$ at $y^* = 0$ in Eq. (9).

It is difficult to obtain the analytical solution for Eqs. (6) and (9), because γ and u^* are complicated functions of y^* . Utilizing the method which combined the Newton-Raphson and Runge-kutta-Gill methods, the eigenvalues and constants have been found numerically for a wide range of both $0.01 \leq Pr \leq 10$ and $10^4 \leq Re \leq 5 \times 10^5$, and are listed in Appendix II.

Entrance Region Nusselt numbers

The typical examples of Nusselt number with x/D_e are shown in Figs. 3 and 4. The difference between the local Nusselt number for symmetric boundary condition and that for asymmetrical boundary condition decreases with increasing Prandtl number. For example, the two values are nearly equal at $Pr = 10$, but at lower Prandtl number, the local Nusselt number for the symmetrical case in the

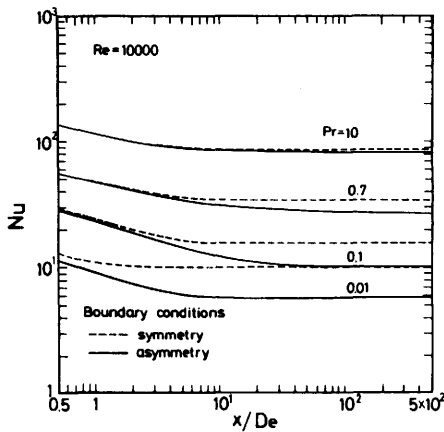


Fig. 3 Local Nusselt number variation for various Prandtl numbers [Re=10000]

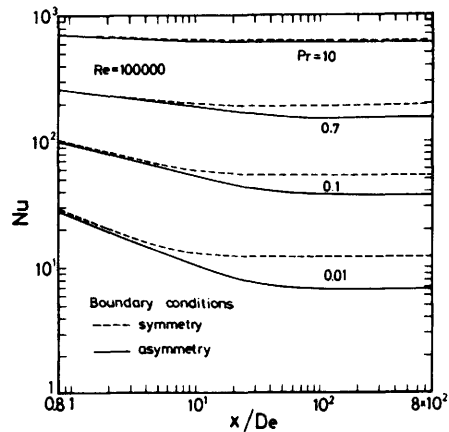


Fig. 4 Local Nusselt number variation for various Prandtl numbers [Re=100000]

fully developed region is some 10% higher than the asymmetrical case. However, these Nusselt numbers are in good agreement when the value of x/De is small.

Thermal Entry Lengths

The thermal entry length is defined in this study to be that distance downstream of the thermal entrance necessary for the local Nusselt number to fall to within 5% of its fully developed value. Calculations of this quantity were carried out for a wide range of Prandtl number and Reynolds number, and the results are shown in Fig. 5. In general, the five percent thermal entry lengths for uniform wall heat flux are slightly longer than those for uniform wall temperature in liquid metal region. For $Pr \geq 0.7$,

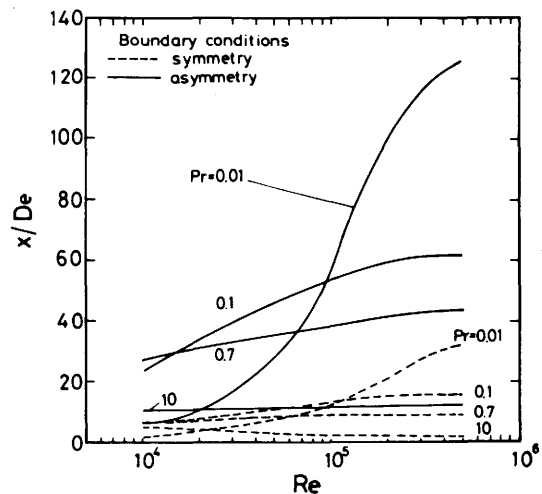


Fig. 5 Five percent thermal entry length

however, the entry lengths for the two cases are essentially identical. Furthermore, the five percent entry lengths for asymmetrical boundary condition are longer than those for symmetrical boundary condition.

Fully Developed Nusselt Numbers

Fig. 6 summarizes the fully developed Nusselt number, and, for comparison, the experimental data of Sparrow et al.¹³⁾ and Duchatelle et al.¹⁴⁾ are included. This figure shows good agreement between this numerical results and the experimental data.

Fig. 7 is a plot of the ratio, $Nu_{\infty}(Q_w)/Nu_{\infty}(T_w)$, for various Prandtl numbers, where $Nu_{\infty}(Q_w)$ shows the fully developed Nusselt number based on uniform wall heat flux and $Nu_{\infty}(T_w)$ shows the fully developed Nusselt number based on uniform wall temperature [see Sakakibara et al.⁴⁾]. It shows the effect of boundary condition on heat transfer rate. As shown in Fig. 7, there is a significant

difference about 13-17%

between the fully developed

Nusselt numbers for uniform

wall heat flux and for uni-

form wall temperature when

the Prandtl number is at

0.01. The difference is

less than 2% when the Prandtl

number is 10.

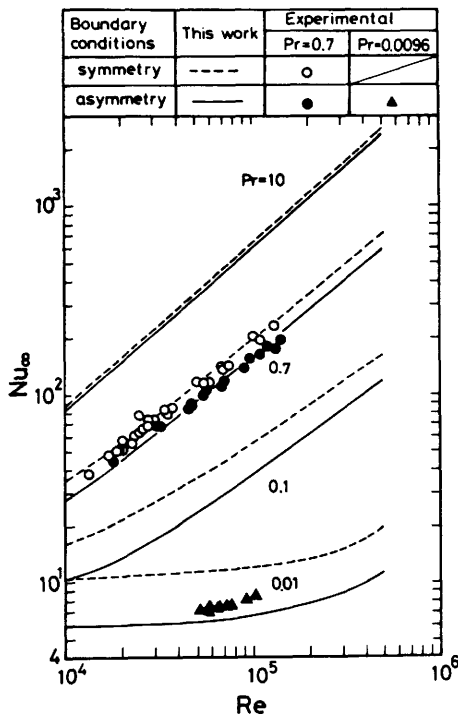


Fig. 6 Fully developed Nusselt number for various Prandtl numbers [Experimental data : O, ● Sparrow et al.¹³⁾ ; ▲ Duchatelle et al.¹⁴⁾]

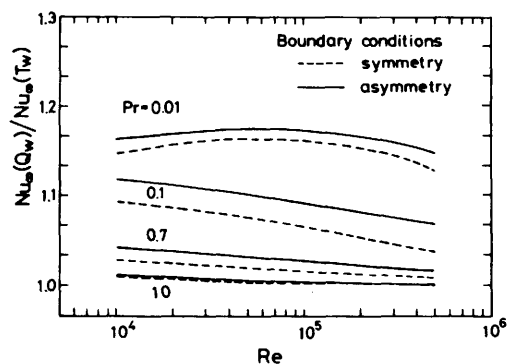


Fig. 7 Ratio of fully developed Nusselt numbers for uniform wall heat flux and for uniform wall temperature

Comparison with Other Models

Fig. 8 shows a comparison of the present solution with that given by Hatton et al.¹⁾ for asymmetrical case. For $Pr=10$, this solution shows lower Nusselt number than that obtained by Hatton et al. This may be attributed to the difference of the values of the fluid velocity, eddy diffusivity and turbulent Prandtl number distributions used in solving the energy equation. For $Pr=0.01$, this solution is in fairly good agreement with Hatton et al.'s solution. In lower Prandtl number, the variation of the local Nusselt number is less affected by the fluid velocity and eddy diffusivity distributions, because the turbulent transport assumptions have less importance in the calculations with fluids of low Prandtl number with dominant molecular heat transfer.

Hatton et al.'s solution is also limited to the value of x/D_e greater than about 1.0. The solution described here is effective to the value of x/D_e greater than about 0.1. The difference is due to the number of the infinite series which form the solution. This is especially true of liquid metal systems. Therefore, Hatton et al.'s solution leads to noticeable difference when the value of x/D_e is small.

The results calculated by Larson et al.²⁾ give quantize Nusselt number than this work.

Comparison with Experimental Results

Fig. 9 shows a comparison between present solution and the experimental data of Larson et al.²⁾ for asymmetric mass transfer. At $Re=30200$, this solution is good agreement with experimental data, but at $Re=11200$, this solution shows lower Nusselt number than the experimental data.

Fig. 10 shows a comparison between present solution and the

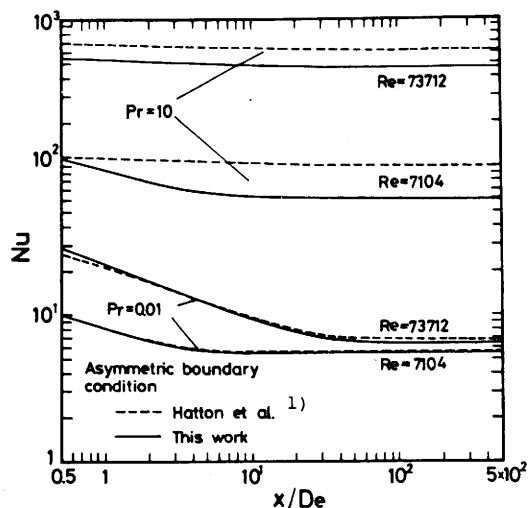


Fig. 8 Comparison of present solution with Hatton et al.'s solution [uniform heat flux on one wall, the other wall insulated]

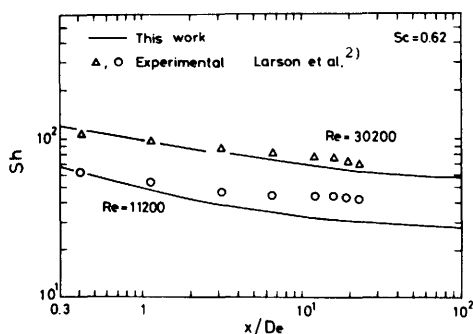


Fig. 9 Comparison of present numerical solution with Larson et al.' mass transfer results [uniform mass flux on one wall, the other wall insulated]

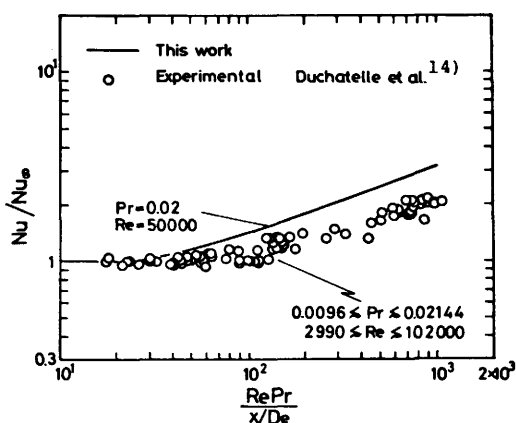


Fig. 10 Comparison of present numerical solution with Duchatelle et al.'s heat transfer results [uniform heat flux on one wall, the other wall insulated]

experimental data of asymmetric heat transfer obtained by Duchatelle et al. ¹⁴⁾ in liquid metal region. The experimental data of Nu/Nu_∞ are conservative in comparison with this solution. The difference can be attributed to the effect of this numerical results obtained without the term of axial heat conduction, and to the experimental data which show higher Nusselt number than empirical results [see Kays et al. ¹⁵⁾] in the fully developed region. However, there is good agreement trend-wise.

CONCLUDING REMARKS

The numerical solution of heat transfer with turbulent flow between parallel plates has been obtained over a wide range of Prandtl number and Reynolds number for uniform heat flux on one wall, the other wall insulated and for uniform heat fluxes on each wall. In this study, the suitable correlations of the fluid velocity, eddy diffusivity and turbulent Prandtl number have been used from a reconsideration of available experimental works.

NOMENCLATURE

C_m	coefficient in Eq. (8)
D_e	$4y_0$
H	distance variation from wall of fully developed temperature profile defined in Eq. (5)
k	thermal conductivity
Nu	local Nusselt number
$Nu_\infty(Q_w)$	fully developed Nusselt number based on uniform wall heat flux
$Nu_\infty(T_w)$	fully developed Nusselt number based on uniform wall temperature
Pr	Prandtl number
Pr_t	turbulent Prandtl number
q	heat flux
Re	Reynolds number ($D_e V/\nu$)
Sc	Schmidt number
Sh	Sherwood number
t	temperature
u	velocity of fluid
u^+	$u/\sqrt{\tau_w/\rho}$
u^*	$u/V = u^+/V^+$
V	mean velocity of fluid
x	coordinate parallel to flat plate
x^*	$(x/D_e)/(RePr)$
Y_m	eigenfunction
y	coordinate normal to flat plate
y_0	half width between parallel plates
y^+	$(y\sqrt{\tau_w/\rho})/\nu$
y^*	$y/y_0 = y^+/y_0^+$

Greek Symbols

α	thermal diffusivity
γ	$1 + Pr(\epsilon_h/\nu)$
ϵ_h	eddy diffusivity for heat
ϵ_v	eddy diffusivity for momentum
θ	dimensionless temperature
ν	kinematic viscosity of fluid
λ_m	eigenvalue
ρ	density of fluid
τ	shear stress

Subscripts

1	fully developed region or boundary of y^+
2	in the entrance region or boundary of y^+
e	entrance
i	at one wall
j	at the other wall
w	solid-liquid interface
∞	fully developed region

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APPENDIX I

$$0 \leq y^+ \leq y_1^+$$

$$\frac{\varepsilon_v}{v} = A(y^+)^3 \quad (1)$$

$$u^+ = \frac{1}{6A^{2/3}} \left(A^{1/3} + \frac{1}{y_0^+} \right) \times \ln \left| \frac{(y^+ + 1/A^{1/3})^3}{(y^+)^3 + 1/A} \right| + \frac{1}{3^{1/2}A^{2/3}} \left(A^{1/3} - \frac{1}{y_0^+} \right) \times \left[\tan^{-1} \left(\frac{2y^+ - 1/A^{1/3}}{3^{1/2}/A^{1/3}} \right) + \frac{\pi}{6} \right] \quad (2)$$

$$y_1^+ \leq y^+ \leq y_2^+$$

$$\frac{\varepsilon_v}{v} = 0.4y^+ \left(1 - \frac{y^+}{y_0^+} \right) - 1 \quad (3)$$

$$u^+ = 2.5 \ln y^+ + 5.5 \quad (4)$$

$$y_2^+ \leq y^+ \leq y_0^+$$

$$\frac{\varepsilon_v}{v} = 0.07y_0^+ \quad (5)$$

$$u^+ = \frac{y^+ - y_2^+}{1 + 0.07y_0^+} \left(1 - \frac{y^+ + y_2^+}{2y_0^+} \right) + 2.5 \ln y_2^+ + 5.5 \quad (6)$$

where A , y_1^+ , y_2^+ are described by

$$A(y_1^+)^3 = 0.4y_1^+ \left(1 - \frac{y_1^+}{y_0^+} \right) - 1 \quad (7)$$

$$\int_0^{y_1^+} \frac{1 - y^+/y_0^+}{1 + A(y^+)^3} dy^+ = 2.5 \ln y_1^+ + 5.5 \quad (8)$$

$$y_2^+ = \frac{1}{2} \left\{ y_0^+ - [0.3(y_0^+)^2 - 10y_0^+]^{1/2} \right\} \quad (9)$$

Appendix II Eigenvalues and constants (Uniform heat flux on one wall, the other wall insulated)

Re=10000

	Pr=10		Pr=0.7		Pr=0.1		Pr=0.01	
H_i	0.01205445		0.03700748		0.09799846		0.1719437	
H_j	-0.0006554534		-0.007835085		-0.03560761		-0.07400190	
m	λ_m	C_m	λ_m	C_m	λ_m	C_m	λ_m	C_m
0	76.08185	0.07691934	21.22473	0.2735388	9.797108	0.4567011	6.753887	0.5354243
1	144.8007	0.04340028	41.11845	0.09785342	19.07537	0.1369241	13.18502	0.1500955
2	208.0624	0.06047872	60.40841	0.06674026	28.30335	0.07058221	19.64844	0.06992915
3	255.8059	0.1041233	78.54533	0.06135258	37.41059	0.04634482	26.11994	0.04072382
4	291.3766	0.1121135	95.20401	0.06110671	46.37053	0.03451032	32.58947	0.02691229
5	333.6039	0.07087323	110.8305	0.05551012	55.19936	0.02728369	39.05298	0.01929038
6	388.6925	0.04066451	126.5968	0.04317849	63.94989	0.02204805	45.50929	0.01462232
7	445.8071	0.03318262	143.3284	0.03058458	72.68906	0.01787319	51.95867	0.01154311
8	498.6041	0.03325740	160.9444	0.02194971	81.46817	0.01450869	58.40206	0.009395158
9	546.8571	0.03165910	178.9678	0.01704231	90.30626	0.01187021	64.84045	0.007830358

Re=50000

	Pr=10		Pr=0.7		Pr=0.1		Pr=0.01	
H_i	0.003022571		0.01099290		0.04246236		0.1589958	
H_j	-0.0001598524		-0.002179435		-0.01447256		-0.07084041	
m	λ_m	C_m	λ_m	C_m	λ_m	C_m	λ_m	C_m
0	149.6932	0.06904129	40.25574	0.2498090	15.43629	0.4281755	6.898903	0.5536081
1	287.6580	0.02524790	78.25683	0.08142366	30.05919	0.1303076	13.48424	0.1543564
2	424.4084	0.01733156	115.5662	0.04866969	44.52868	0.06952235	20.09689	0.07181594
3	554.7968	0.01620178	151.6743	0.03774243	58.71379	0.04704747	26.71104	0.04174362
4	679.6596	0.01782363	186.4468	0.03325843	72.59086	0.03586127	33.31799	0.02746794
5	799.1579	0.02059825	220.2715	0.02987650	86.25019	0.02836078	39.91684	0.01952034
6	916.9268	0.02470323	253.8950	0.02668932	99.85830	0.02262532	46.51059	0.01460712
7	1033.018	0.03101632	287.8917	0.02370124	113.5661	0.01801314	53.10359	0.01133447
8	1145.198	0.04162280	322.2841	0.02177948	127.4318	0.01454724	59.69966	0.009043021
9	1248.299	0.05362350	356.6483	0.02104348	141.4150	0.01211305	66.30087	0.007378191

Re=100000

	Pr=10		Pr=0.7		Pr=0.1		Pr=0.01	
H _i	0.001656176		0.006347701		0.02704430		0.1465475	
H _j	-0.00008742994		-0.001199661		-0.008653193		-0.06508773	
m	λ_m	C _m	λ_m	C _m	λ_m	C _m	λ_m	C _m
0	202.5791	0.06817541	54.17082	0.2396926	19.91440	0.4071076	7.196646	0.5521690
1	389.6227	0.02380120	105.3829	0.07741992	38.78799	0.1255323	14.07584	0.1542948
2	575.0036	0.01539475	155.5856	0.04642928	57.39489	0.06918365	20.97366	0.07240976
3	751.9318	0.01300382	204.0678	0.03560511	75.53610	0.04830187	27.86089	0.04252978
4	922.1839	0.01262632	250.6826	0.03106395	93.18063	0.03801031	34.72938	0.02826299
5	1086.910	0.01231201	296.0666	0.02701750	110.4930	0.03045611	41.58179	0.02021838
6	1252.125	0.01217886	341.3546	0.02336269	127.7629	0.02442697	48.42640	0.01517622
7	1419.389	0.01218134	387.3703	0.01976806	145.2291	0.01930461	55.27286	0.01176537
8	1588.665	0.01331002	434.1302	0.01743746	162.9518	0.01558384	62.12848	0.009358651
9	1755.626	0.01568569	481.0702	0.01604237	180.8313	0.01296979	68.99614	0.007605365

Re=500000

	Pr=10		Pr=0.7		Pr=0.1		Pr=0.01	
H _i	0.0004069628		0.001694719		0.008233107		0.08735625	
H _j	-0.00002079830		-0.0002685642		-0.002163020		-0.03572005	
m	λ_m	C _m	λ_m	C _m	λ_m	C _m	λ_m	C _m
0	414.8788	0.06426004	110.3796	0.2126229	39.48447	0.3412475	9.749619	0.5120618
1	799.3795	0.02216145	215.1112	0.06977543	76.95532	0.1102148	19.06570	0.1490727
2	1179.595	0.01358304	317.4526	0.04172853	113.5922	0.06460005	28.31161	0.07537002
3	1541.319	0.01104204	415.8272	0.03290181	148.8587	0.04929047	37.42260	0.04799827
4	1888.899	0.009746112	510.0067	0.02840219	182.7052	0.04115397	46.39656	0.03427524
5	2226.092	0.008670070	601.6404	0.02479135	215.7295	0.03450842	55.28463	0.02556366
6	2566.425	0.007389079	693.4149	0.02072299	248.8649	0.02782619	64.16672	0.01943682
7	2913.129	0.006380021	787.0295	0.01733378	282.6946	0.02231510	73.11097	0.01487376
8	3266.121	0.005682783	882.2364	0.01479578	317.1414	0.01830046	82.14363	0.01161590
9	3617.564	0.005422744	977.6440	0.01342707	351.7368	0.01586307	91.24747	0.009304765